GENERAL CERTIFICATE OF EDUCATION BOARD
General Certificate of Education Examination

Pure Maths With Stats 2
0770/2

JUNE 2023 **ADVANCED LEVEL**

Subject Title	**Pure Mathematics With Statistics**
Paper No.	**Paper 2**
Subject Code No.	**0770**

THREE HOURS.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae Booklets published by the GCE Board are allowed.

In calculations, you are advised to show all the steps in your working, giving the answer at each stage.

Calculators are allowed.

Start each question on a fresh page.

Turn Over

1. (i) A polynomial P is defined on the set $\mathbb{R}$ of real numbers by
$$P(x) = 4x^3 + 4x^2 - 11x - 6.$$
Given that $(2x - 3)$ is a factor of $P(x)$.
(a) factorise $P(x)$ completely.
(b) solve the inequality $P(x) < 0$. **(5, 3 marks)**

(ii) The roots of the quadratic equation $3x^2 - 7x + 1 = 0$ are α and β.

Find the value of the expression $\left(2\alpha + \frac{1}{\beta}\right) + \left(2\beta + \frac{1}{\alpha}\right)$. **(4 marks)**

---

2. (i) Solve for x, the equation $\log_2(x - 3) + \log_2 x = 2$. **(3 marks)**

(ii) Express $f(\theta)$, where
$$f(\theta) = \frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta,$$
in the form $r\cos(\theta - \lambda)$, where $r > 0$ and λ an acute angle.
Hence find the maximum value of the expression $\frac{2}{4 + f(\theta)}$. **(7 marks)**

---

3. (i) A function f is defined on $\mathbb{R}$ by
$$f(x) = \frac{3x}{5x - 2}.$$
(a) State the domain of f.
Show that f is
(b) injective,
(c) monotone decreasing. **(1, 3, 3 marks)**

(ii) Find the range of real values of the function $y = \frac{3x^2}{4x - 1}, x \in \mathbb{R}, x \neq \frac{1}{4}$. **(3 marks)**

---

4. The table below shows corresponding values of x and y which approximately satisfy a relation of the form
$$y = ax^2 + bx,$$

x	2.0	2.5	3.0	3.5	4.0	4.5	5.0
y	28.0	31.3	33.0	33.3	32.0	29.3	25.0

By drawing a suitable linear graph, determine, correct to one decimal place, the values of the constsnts a and b. **(9 marks)**

---

5. (i) Given the complex number $Z = \frac{5 + i}{2 + 3i}$,
(a) express Z in the form $x + yi$ where x and y are real constants.
(b) show that $z = \sqrt{2}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)$.
(c) By equating the form in (a) to that in (b) show that $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$. **(3, 4, 2 marks)**

(ii) The line $y = 3 - 2x$ is a tangent to a circle whose centre is at the point $(3, 2)$.
Find the equation of this circle. **(3 marks)**

---

6. (i) The equation of a curve C, is given by

$$y = \frac{x}{3}(x^2 - 4).$$

Find,

(a) $\frac{dy}{dx}$ at the point $P(-1, 1)$,

(b) the equation of the tangent T, to C at the point $P(-1, 1)$,

(c) the coordinates of the point Q, where the tangent T meets C again. **(2, 2, 3 marks)**

(ii) Given that $f(x) = \frac{3x}{(x-1)(x+2)}$, $x \in \mathbb{R} - \{-2,1\}$,

(d) express $f(x)$ into partial fractions.

(e) find $\int f(x)\,dx$. **(6 marks)**

---

7. (i) Evaluate

$$\sum_{r=0}^{40}(3r + 1)$$ **(3 marks)**

(ii) A class of 9 students and a teacher is out on a field trip. On arrival at a car park they found out that there is only one taxi available to take them to their destination. Given that the taxi can carry only four passengers, find the number of ways that the 6 who will treck can be selected if:

(a) the teacher must go by taxi,

(b) the teacher will not go by taxi. **(3, 3 marks)**

---

8. (i) Taking $x = -0.8$ as the first approximation to a root of the equation

$$e^x - \ln(1 + x^2) = 0,$$

use the Newton-Raphson method to find, to three decimal places, a second approximation to the root of the equation. **(5 marks)**

(ii) Prove, by Mathematical induction, that $3^n + 1$ is even for all positive integral values of n. **(5 marks)**

---

9. (i) Given the plane Π; $x - 2y - z = 4$ and the line L: $\frac{x-2}{3} = \frac{y+4}{1} = \frac{z-1}{-1}$,

find,

(a) the sine of the acute angle between Π and L,

(b) the vector equation of the plane which is normal to the L and contains the point (2, 3, 1). **(3, 3 marks)**

(ii) A linear transformation of points in 3- dimensional space is defined by the matrix **M**, where

$$\mathbf{M} = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$$

Under this transformation, find the image of the points on the line

$$\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{-1}.$$ **(3 marks)**

---

10. (i) Given that

$$2y = x^2 + \sin y,$$

show that $(\cos y - 2)\frac{d^2y}{dx^2} - (\sin y)\left(\frac{dy}{dx}\right)^2 + 2 = 0.$ **(4 marks)**

(ii) A function $h: \mathbb{R} - \{-2\} \to \mathbb{R}$ is defined by

$$h(x) = \frac{x}{x+2}.$$

Sketch the curve $y = h(x)$. **(5 marks)**
