FURTHER MATHEMATICS PAPER 3
0775

GOVERNMENT BILINGUAL HIGH SCHOOL YAOUNDE

MOCK GCE

APRIL 2021 ADVANCED LEVEL

Subject Title	**Further Mathematics**
Paper Number	**Paper 3**
Subject Code	**0775**

Two and a half hours

Answer ALL questions.

For your guidance, the approximate mark allocation for parts of each question is indicated.

Mathematical formulae booklets and tables published by the Board and noiseless nonprogrammable electronic calculators are allowed.

In calculations, you are advised to show all the steps in your working, giving answers at each stage.

1. Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acts at points with position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ respectively where

$$\mathbf{F}_1 = (\mathbf{i} - 3\mathbf{j} + 3\mathbf{k})N \;, \mathbf{r}_1 = (3\mathbf{i} + \mathbf{j} - 2\mathbf{k})m$$

$$\mathbf{F}_2 = (2\mathbf{i} + 2\mathbf{j} - 12\mathbf{k})N \;\; \mathbf{r}_2 = (\mathbf{i} + 3\mathbf{j} + \mathbf{k})m$$

(a) Show that the lines of action of $\mathbf{F}_1$ and $\mathbf{F}_2$ intersect at the point with position vector $(2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k})m$ **(5marks)**

(b) The forces $\mathbf{F}_3 = (\mathbf{i} + 3\mathbf{j} - \mathbf{k})N$ and $\mathbf{F}_4$ acts through the points with position vector $(\mathbf{i} + \mathbf{j} - \mathbf{k})m$ so that the system of four forces reduces to a couple, **C**.

(i) Find the force $\mathbf{F}_4$. **(2 marks)**

(ii) Find the magnitude of the couple **C**. **(5 marks)**

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2. A particle P, of mass mkg moves in a straight line. At time t seconds the displacement of P from a fixed point O on the line is x metres and P is moving with velocity $\dot{x}\ ms^{-1}$. Throughout the motion, two horizontal forces act on P; a force of magnitude $4mn^2|x|$ *newtons* directed towards O, and a resistance force of magnitude $2mk|\dot{x}|$ *newtons*, where n and k are positive constants.

(a) Show that $\ddot{x} + 2k\dot{x} + 4n^2x = 0$ **(2 marks)**

(b) In one case $k = n$, when $t = 0, x = a$ and $\dot{x} = 0$.

(i) Show that

$$x = e^{-nt}\left[a\cos\sqrt{3}nt + \left(\frac{\sqrt{3}a}{3}\right)\sin\sqrt{3}nt\right]$$ **(6 marks)**

(ii) Show that P passes through O when $\tan\left(\sqrt{3}nt\right) = -\sqrt{3}$ **(2 marks)**

(c) In a different case $k = 2n$

(i) Find a general solution for x at time t seconds. **(2 marks)**

(ii) Hence state the type of damping which occurs **(1 mark)**

3.

(i) Given that y satisfies the differential equation $(x+1)\dfrac{dy}{dx} = 2x - y$.

(a) Find a solution to the above differential equation as a Taylor series of ascending powers of $(x-1)$ up to and including the terms in $(x-1)^3$ given that $y = 0.5$ when $x = 1$. **(4 marks)**

(b) Use the approximation $y_{n+1} = y_{n-1} + 2h\left(\dfrac{dy}{dx}\right)_n$ and a step length of 0.2 to find the value of y when $x = 1.4$. **(5marks)**

(ii) The table below gives the speed $v\ ms^{-1}$ of a particle in a straight line between two points A and B. At time $t = 0.2s$ the particle is at A, and at time $t = 1s$ the particle is at B.

$t(s)$	0.2	0.4	0.6	0.8	1.0
$v(ms^{-1})$	6.03	4.04	2.71	1.82	1.22

Using Simpson rule, estimate to two decimal places, the distance AB. **(4marks)**

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4. Two smooth spheres A and B have equal radii and masses $2m$ kg and m kg respectively. The spheres are moving on a smooth horizontal plane. The sphere A has velocity $(3\mathbf{i}+\mathbf{j})\,ms^{-1}$ when it collides with the sphere B, which has velocity $(2\mathbf{i}-5\mathbf{j})\,ms^{-1}$. Immediately after the collision, the velocity of the sphere B is $(2\mathbf{i}+\mathbf{j})\,ms^{-1}$

(a) Find the velocity of A immediately after the collision. **(2 marks)**

(b) Show that the impulse exerted on B in the collision is $(6m\mathbf{j})\,Ns$ **(2 marks)**

(c) Find the coefficient of restitution between the two spheres **(3 marks)**

(d) After the collision, each sphere moves in a straight line with constant speed. Given that the radius of each sphere is 0.05 m, find the time taken, from the collision, until the centers of the spheres are 1.10 m apart.

(6 marks)

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5. A particle Q of mass m falls under gravity in a medium whose resistance to motion is of magnitude mkv^2, where v is the speed of Q and k is a positive constant. Given that the maximum speed of Q is u.

(a) Show that $u=\sqrt{\frac{g}{k}}$ **(3marks)**

Given that Q is projected vertically upward with speed $v(>u)$.

(b) Show that the speed of Q is equal to u when the height of Q above the point of projection is

$$\frac{u^2}{2g}\ln\left(\frac{1}{2}+\frac{v^2}{2u^2}\right)$$ **(6marks)**

(c) Find in terms of u, v and g the time taken, t for the speed of A to decrease from u to v.

(4marks)

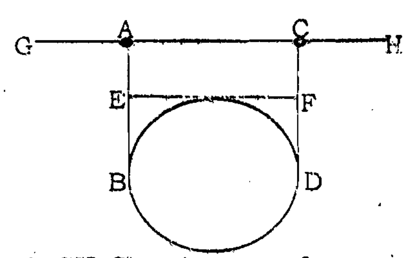
6.

(i) A uniform circular disc has mass 20m and radius a

(a) Prove, by integration that the moment of inertial of the disc about an axis through its centre and perpendicular to the plane of the disc is $10ma^2$ **(4marks)**

(b) Hence determine the moment of inertial of the disc about a diameter, starting clearly any theorems that you use. **(3 marks)**

(ii) A shop sign consists of a uniform circular disc of mass 20 m and radius a, which is rigidly fixed to three rods AB, CD and EF. Each rod is of mass $20m$ and length $2a$. The rods are attached so that BD is a diameter of the disc; EF is a tangent to the disc, with A vertically above E and C vertically above F.
The sign is suspended on a fixed horizontal pole GH by means of two small light rings which are attached to the sign at A and C, as shown in the diagram.



The sign can swing freely about the pole GH. Show in terms of m and a, that the moment of inertial of the sign about GH is $\frac{277}{3}ma^2$. **(6 marks)**

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7.

(i) A particle P moves in a plane. At time t seconds, its polar coordinate (r,θ) are given by $r = at^2, \theta = \frac{1}{3}t^4$ where a is a constant. Show that the speed of P at time $t = 2s$ is $4a\sqrt{17}$. **(4marks)**

(ii) A bead B, moves along a smooth wire in the form of a curve whose polar equation is $r = \lambda(1+\cos\theta)$, where $\lambda > 0$, in such a way that the transverse component of the acceleration is zero. Show that the radial component of the acceleration in terms of r and θ is $(\lambda - 2r)\left(\frac{d\theta}{dt}\right)^2$ **(7marks)**

8.

(i) Of all the torches produced by a certain factory, 10 % are faulty. Ten torches are chosen at random.

(a) Find the probability that at most two torches are faulty. **(3marks)**

(b) The probability that a box of n torches contains no torch that is faulty is less than 0.1. Find the range of possible values of n. **(4marks)**

(ii) The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} kx(1-x), & 0 \le x \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find:

(a) the value of k

(b) the mode of X. **(5marks)**

END