Pure Mathematics With Mech 2
0765/2

CAMEROON GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

JUNE 2019 **ADVANCED LEVEL**

Subject Title	**Pure Mathematics With Mechanics**
Paper No.	**Paper 2**
Subject Code No.	**0765**

Three hours

Full marks may be obtained for answers to ALL questions.

Mathematical formulae Booklets published by the Board are allowed.

In calculations, you are advised to show all the steps in your working, giving the answer at each stage.

Calculators are allowed.

Start each question on a fresh page.

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1. (i) The polynomial $P(x)$ is such that $P(x) = a\,x^3 - 3\,x^2 + b\,x + 6$. Given that $(x+2)$ is a factor of $P(x)$ and that when $P(x)$ is divided by $(x-1)$ the remainder is -6.
   (a) Find, the values of the constants a and b.
   (b) Hence, solve the equation $P(x) = 0$. **(7 marks)**

   (ii) Two statements p and q are given by
   p: Comfort studies hard.
   q: She will pass the examination.
   Write out the following propositions in simple English.
   (c) $p \Rightarrow q$
   (d) $\sim p \Rightarrow \sim q$
   (e) $\sim(p \Rightarrow q)$ **(3 marks)**

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2. Two variables X and Y are related by an equation of the form $Y = \log_{10}(m\,X + c)$.
   The table below gives the corresponding values of X and Y.

X	1	2	3	4	5	6
Y	0.857	0.924	0.982	1.033	1.079	1.121

   (a) Draw a linear graph plotting the values of 10^Y against X.
   (b) Use your graph to estimate, to the nearest whole number, the values of m and c.
   (8 marks)

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3. (i) A function f is defined by
   $$f : x \mapsto \frac{2\,x+1}{x-4}, \; x \in \mathbb{R}, \; x \neq 4.$$
   (a) Show that f is injective.
   (b) Find the inverse function $f^{-1}(x)$, stating its domain. **(6 marks)**

   (ii) A relation $\mathcal{R}$ is defined on the set of integers by: $a\,\mathcal{R}\,b \Leftrightarrow a + b = 2m + 1$, where m is an integer.
   Show that $\mathcal{R}$ is not an equivalence relation. **(3 marks)**

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4. (i) Given that $(k+5)x^2 - 10\,x + 2k\,x = 9k$ is a quadratic equation, find the value(s) of the constant k for which the roots are equal. **(6 marks)**

   (ii) There are 6 girls and four boys in a class. 3 students are to be chosen at random so as to be awarded a scholarship. In how many ways can this be done if at least 1 boy and 1 girl must be in the selection. **(3 marks)**

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5. (i) Given that $f(x) = x^3 - 4\,x^2 - x - 12$. Verify that $f(x) = 0$ has a root in the interval $4 < x < 5$. Taking $x = 4.5$ as a first approximate root of the equation $f(x) = 0$, use the Newton - Raphson procedure to find a second approximation to the root of the equation, giving the answer to two decimal places. **(7 marks)**

   (ii) The sum of the first n terms of a sequence is given by $S_n = 2\,n^2 + n$.
   Find,
   (a) the tenth term of the sequence.
   (b) the n^{th} term of the sequence. **(5 marks)**

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6. (i) Solve the differential equation $\left(x^2 - 1\right)\dfrac{dy}{dx} + 2\,y = 0$, given that $y = 3$ when $x = 2$
   Express the answer in the form $y = f(x)$. **(5 marks)**

   (ii) Given that $f(x) = \dfrac{2\,x+1}{x-4}$. Sketch the graph of $y = f(x)$, showing clearly all its intercepts and the behaviour of the curve as it approaches its asymptotes. **(6 marks)**

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7. The vector equations of two lines $\mathcal{L}_1$ and $\mathcal{L}_2$ are given by
   $$\mathcal{L}_1 : \mathbf{r} = \mathbf{i} - \mathbf{j} + 3\,\mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + \mathbf{k})$$
   $$\mathcal{L}_2 : \mathbf{r} = 2\,\mathbf{i} + a\,\mathbf{j} + 6\,\mathbf{k} + \mu(2\,\mathbf{i} + \mathbf{j} + 3\,\mathbf{k}),$$
   where a, λ and μ are real constants. Given that $\mathcal{L}_1$ and $\mathcal{L}_2$ intersect, find
   (a) the value of the constant a. **(5 marks)**

(b) the position vector of the point of intersection of $\mathcal{L}_1$ and $\mathcal{L}_2$. **(2 marks)**

(c) the cosine of the acute angle between $\mathcal{L}_1$ and $\mathcal{L}_2$. **(3 marks)**

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8. (i) The function f is defined by $f(x) = \dfrac{2}{x^2 - 1}$,

(a) express f(x) in partial fractions.

Hence,

(b) show that $\displaystyle\int_3^5 f(x)\,dx = \ln\left(\frac{4}{3}\right)$. **(6 marks)**

(ii) Find $\displaystyle\int \cos^3 x \sin^3 x\,dx$. **(4 marks)**

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9. (i) Given that $f(\theta) = \sin\theta - \sqrt{3}\cos\theta$,

(a) express $f(\theta)$ in the form $r\sin(\theta - \lambda)$, where $r > 0$ and $0 < \lambda < \dfrac{\pi}{2}$.

Hence,

(b) find the maximum and minimum values of $\dfrac{1}{f(\theta) + 3}$.

(ii) Find the intervals for which the function $h(x) = x^3 - 3x$ is strictly

(a) increasing

(b) decreasing. **(6, 5 marks)**

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10. (i) Given that $z = 1 - i\sqrt{3}$, express z in the form $r(\cos\theta + i\sin\theta)$.

Hence write down z^7 in the form $r e^{i\theta}$. **(3 marks)**

(ii) Given that $\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 2 & 3 & 0 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix}$ are two matrices,

find the matrix product **AB** and **BA**.

State the relationship between **A** and **B**.

Find, also, the matrix product **BM**, where $\mathbf{M} = \begin{pmatrix} 8 \\ -7 \\ 1 \end{pmatrix}$.

Hence,

solve the system of equations

$$\begin{cases} x - y + z = 8 \\ 2y - z = -7 \\ 2x + 3y = 1 \end{cases}.$$ **(8 marks)**
