FURTHER MATHEMATICS PAPER 2
0775

CAMEROON GENERAL CERTIFICATE OF EDUCATION BOARD

General Certificate of Education Examination

JUNE 2019 **ADVANCED LEVEL**

Subject Title	**Further Mathematics**
Paper No.	**2**
Subject Code No.	**0775**

THREE HOURS

Answer ALL 10 questions.

For your guidance, the approximate mark allocation for parts of each question is indicated.

Mathematical formulae and tables published by the Board , and noiseless non-programmable electronic calculators are allowed.

In calculations, you are advised to show all the steps in your working, giving your answer at each stage.

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1. Find the complementary function of the differential equation

$$2\,\frac{d^2y}{dx^2} - \frac{dy}{dx} - 3\,y = 5\,e^{-x},$$

in the form $y = f(x)$. **(3 marks)**

Hence find the particular integral and the general solution in the form $y = f(x)$. **(4 marks)**

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2. (a) Express $f(x)$ in partial fractions where

$$f(x) = \frac{2\,x^3 + x + 2}{(x^2+1)\,(x+1)(x-2)},\quad x \neq -1, 2.$$ **(4 marks)**

Hence, or otherwise, show that

$$\int_0^1 f(x)\,dx = -\frac{1}{12}[13\ln 2 + \pi]$$ **(4 marks)**

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3. (a) Solve the equation

$$\tanh^{-1}\left(\frac{x-2}{x+1}\right) = \ln 2.$$ **(4 marks)**

(b) Show that the set $\{1, 2, 4, 8\}$ under $\times_{15}$, multiplication mod 15, forms a group. **(4 marks)**

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4. (a) Given that the matrix M is defined by

$$M = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$$

Prove by induction that

$$M^n = \begin{pmatrix} 3^n & 3^n - 2^n \\ 0 & 2^n \end{pmatrix},\ \text{for all } n \geqslant 1.$$ **(4 marks)**

(b) A curve is given by the parametric equations

$$x = t^2,\ y = t\left(1 - \frac{1}{3}\,t^2\right),\ 0 \leqslant t \leqslant \sqrt{3}.$$

Show that the length of the curve is $2\sqrt{3}$. **(3 marks)**

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5. Show that the curve with polar coordinates (r, θ) where

$$r = \frac{4}{-3 + 3\sin\theta},\ \theta \neq n\pi + (-1)^n\frac{\pi}{2},\ n \in \mathbb{Z},$$

is a parabola, $\mathcal{P}$, in the (x, y) plane. **(2 marks)**

Show that the point $\left(2, -\frac{5}{6}\right)$ lies on $\mathcal{P}$ and find the equation of the tangent to $\mathcal{P}$ at this point.

(3 marks)

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6. (a) By the use of the Chinese Remainder Theorem, or otherwise, solve the system of congruences

$$x \equiv 3 \pmod 4$$
$$x \equiv 4 \pmod 7$$ **(5 marks)**

(b) A complex number z is defined by $z = \frac{1}{2}(\cos\theta + i\sin\theta)$, such that

$$z^n = \frac{1}{2^n}(\cos n\theta + i\sin n\theta).$$

Using the De Moivre's theorem, or otherwise, show that

i) $\sum_{r=0}^{\infty} \frac{1}{4^r}\sin 2r\theta$ is a convergent geometric progression. **(1 mark)**

ii) $\sum_{r=0}^{\infty} \frac{1}{4^r}\sin 2r\theta = \frac{4\sin 2\theta}{17 - 16\cos 2\theta}$. **(4 marks)**

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7. A transformation, f, on a complex plane is defined by

$$z' = 2\,z + 3 - 4\,i.$$

(i) Find the image of the point $z = 2 - i$. **(1 mark)**

(ii) Determine the invariant point of f in the form $a + i\,b,\ a, b \in \mathbb{R}$. **(2 marks)**

(iii) Show that f is a similarity transformation (similitude), stating its radius. **(2 marks)**

(iv) Give the geometrical interpretation of f. **(1 mark)**

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8. Given two vectors

$$\mathbf{a} = \alpha\,\mathbf{i} - \mathbf{j} - 4\,\mathbf{k} \text{ and } \mathbf{b} = 3\,\mathbf{i} + 2\,\mathbf{j} + (1 + 2\beta)\mathbf{k},\ \alpha, \beta \in \mathbb{Z},$$
$$\mathbf{a} \times \mathbf{b} = 3\,\mathbf{i} - 21\,\mathbf{j} + 6\,\mathbf{k}.$$

(i) Calculate the values of the real constants α and β. **(3 marks)**

(ii) By using the values of α and β, state the vectors **a** and **b**. **(1 mark)**

(iii) Show that **a** and **b** are linearly independent. **(2 marks)**

(iv) Find the Cartesian equation of the plane containing **a** and **b**. **(2 marks)**

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9. A function, f, is defined by

$$f(x) = \frac{1}{(1 + e^x)^2}.$$

(i) Find the domain of f. **(1 mark)**

(ii) Find the intercept(s) of the curve $y = f(x)$. **(2 marks)**

(iii) Find

$$\lim_{x \to -\infty} f(x) \text{ and } \lim_{x \to +\infty} f(x),$$

and state the asymptotes of the curve $y = f(x)$. **(3 marks)**

(iv) Determine $f'(x)$ and $f''(x)$. **(4 marks)**

(v) Prove that there are no turning points. **(2 marks)**

(vi) Prove, also, that $\left(-\ln 2, \frac{4}{9}\right)$ is the only point of inflexion. **(2 marks)**

(vii) Obtain the intervals on which f is concave up and intervals on which f is concave down. **(2 marks)**

(viii) Obtain a variation table for f. **(2 marks)**

(ix) Sketch the curve, $y = f(x)$. **(2 marks)**

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10. Two sequences, (u_n) and (v_n), for $n \in \mathbb{N}$ are defined as follows:

$$\begin{cases} u_0 = 3 \\ u_{n+1} = \frac{1}{2}(u_n + v_n) \end{cases} \text{ and } \begin{cases} v_0 = 4 \\ v_{n+1} = \frac{1}{2}(u_{n+1} + v_n) \end{cases}.$$

(i) Calculate $u_1,\ v_1,\ u_2$ and v_2. **(5 marks)**

(ii) Another sequence (w_n) is defined by

$$w_n = v_n - u_n,\ \forall n \in \mathbb{N}.$$

(iii) Show that (w_n) is a convergent geometric sequence. **(2 marks)**

(iv) Express (w_n) as a function of n and obtain its limit. **(4 marks)**

(v) Study the sense of variation (monotony) of (u_n) and (v_n).
What can you deduce? **(4 marks)**

(vi) Consider another sequence, (t_n), defined by

$$t_n = \frac{u_n + 2\,v_n}{3},\ \forall n \in \mathbb{N}.$$

(vii) Show that (t_n) is a constant sequence. **(2 marks)**

(viii) Hence, obtain the limits of the sequences (u_n) and (v_n). **(3 marks)**

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