



Scholars
Program



AIMS

African Institute for
Mathematical Sciences
CAMEROON



AIMS TEACHER TRAINING PROGRAM (TTP) IN PARTNERSHIP WITH
MASTERCARD FOUNDATION AND THE GOVERNMENT OF CAMEROON

MATHEMATICS OLYMPIAD

LEVEL: NATIONAL

DATE: 15TH MAY 2021

DURATION OF PAPER: 2 hours

CANDIDATES: UPPER SIXTH

PART B

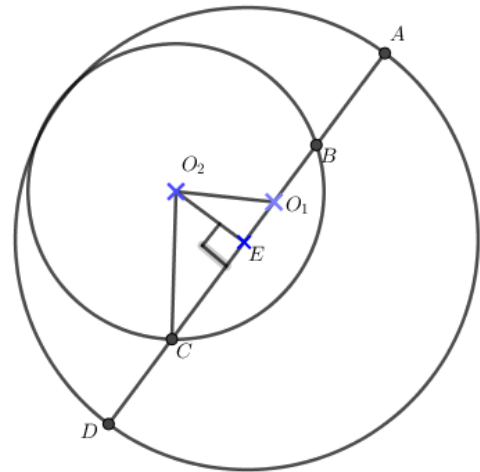
INSTRUCTIONS TO CANDIDATES:

- Mobile phones are **NOT ALLOWED** in the examination room
- You should attempt to answer all questions.
- You are reminded of the necessity for orderly presentation and good English in your work.
- In calculations, you are advised to show all steps in your work, and show answers at each stage
- Non-programmable electronic calculators are allowed
- Graph paper will be provided

INSTRUCTIONS: ANSWER ALL FOUR QUESTIONS IN THIS SECTION. EACH QUESTION CARRIES 15 MARKS

- 1 (i) Find the number of solutions in the set of positive integers of the following equations:
- $x + y = 14$
 - $x + y + z = 5$
- (ii) Solve simultaneously, the linear congruences:
- $$\begin{cases} x \equiv 1 \pmod{3} \\ x \equiv 5 \pmod{8} \\ x \equiv 11 \pmod{17} \end{cases}$$
- (iii) Show that the equation $x^2 - y^2 = 74$ has no integer solution

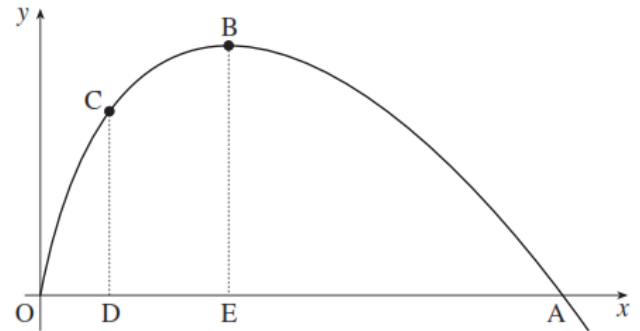
- 2 Two circles are internally tangent. A line passing through the center(O_1) of the larger circle intersects it at the points A and D. The same line intersects the smaller circle with center(O_2) at the points B and C as shown below. Given that $|AB|:|BC|:|CD| = 3:7:2$, find the ratio of the radii of the circles.



3

The Figure beside shows the curve of $y = 2x - x \ln x$, where $x > 0$

The curve crosses the x -axis at A, and has a turning point at B. The point C on the curve has x -coordinate 1. Lines CD and BE are parallel to the y -axis.



- i) Find the x -coordinate of A, giving your answer in terms of e .
- ii) Find the exact coordinates of B.
- iii) Show that the tangents at A and C are perpendicular to each other.
- iv) Using integration by parts, show that

$$\int x \ln x dx = \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C$$

Hence find the exact area of the region enclosed by the curve, the x -axis and the lines CD and BE

4

i) Given that $I_n = \int_0^{\frac{\pi}{2}} e^{-x} \cos^n x dx$, where $n \geq 2$, prove that:

a) $I_n = 1 - n \int_0^{\frac{\pi}{2}} e^{-x} \sin x \cos^{n-1} x dx$

b) $(n^2 + 1)I_n = 1 + n(n - 1)I_{n-2}$

c) $I_6 = \frac{263 - 144e^{-\frac{\pi}{2}}}{629}$

ii) Test whether the series $\sum_{n=0}^{\infty} \left(\frac{2^{n-1}}{4+n}\right)$ converges or not.