AIMS TEACHER TRAINING PROGRAM (TTP) IN PARTNERSHIP WITH MASTERCARD FOUNDATION AND THE GOVERNMENT OF CAMEROON

# MATHEMATICS OLYMPIAD

## LEVEL: NATIONAL

**DATE: 15TH MAY 2021**

**DURATION OF PAPER: 2 hours**

**CANDIDATES: UPPER SIXTH**

## PART A

**INSTRUCTIONS TO CANDIDATES:**

- Mobile phones are **NOT ALLOWED** in the examination room
- You should attempt to answer all questions.
- Each MCQ is 1 mark. Marks for each structural question is indicated.
- You are reminded of the necessity for orderly presentation and good English in your work.
- In calculations, you are advised to show all steps in your work, and show answers at each stage
- Non-programmable electronic calculators are allowed
- Graph paper will be provided

**Instructions for MCQs:**
Each of the following question has four suggested answers. Copy the question number and write down the letter corresponding to the correct answer.

**1** Given that *A*, *B* and *C* are invertible matrices. Then $(ABC)^{-1}$ is:

A)$A^{-1}B^{-1}C^{-1}$; B) $C^{-1}B^{-1}A^{-1}$

C) $C^{-1}A^{-1}B^{-1}$; D) $A^{-1}C^{-1}B^{-1}$

**2** $\int_0^{\frac{\pi}{2}} \cos 3x \sin 3x \, dx =$

A) $\frac{1}{3}$; B) 0; C) $\frac{1}{6}$; D) $-\frac{1}{6}$

**3** The gradient of the curve $\begin{cases} x = sin\theta \\ y = cos\theta \end{cases}$ at $\theta = \frac{\pi}{4}$ is

A)1; B) -1; C) $\frac{1}{\sqrt{2}}$; D) $-\frac{1}{\sqrt{2}}$

**4** The radius of the circle with equation $x^2 + y^2 - 10x + 12y + 41 = 0$ is

A) $5\sqrt{2}$; B) $3\sqrt{5}$; C) $2\sqrt{5}$; D) $4\sqrt{5}$

**5** A relationship between $(y-2)$ and $(x-1)$ is given by $(y-2) = a\,b^{(x-1)}$. With a suitable linear relationship between $(y-2)$ and $(x-1)$, the intercept of the straight line in the axis of $\log(y-2)$ is:

$A)\log b$; B) $2\log b$; C) $2\log a$; D) $\log a$

**6** The point (3, 10) is on the graph of the function $f(x)$ and $f'(3) = 114$. Given that $x = 3$ is a first approximate root of the equation $f(x) = 0$, using the Newton-Raphson procedure, a value of a second approximation to the root of the equation is:

A) 3.09; B) 2.92; C) 2.91; D) 3.08

**7** There are 6 black cards and 4 red cards on a table. In how many ways can 4 cards be selected from the table if the number of colours in the selection must be equal?

A)15 B) 24; C) 90; D) 60

**8** The equations of the asymptotes of the graph of the curve $y = \frac{x-2}{x^2-4x+3}$ are :

A) $x = 1, x = -3, y = 1$; B) $x = -1, x = 3, y = 0$

C) $x = 1, x = 3, y = 0$; D) $x = 1, x = 3, y = 1$

**9** The length of the radius of the sphere $x^2 + y^2 z^2 + 22x - 4y = 0$ is

A) 3.16; B) 3.38; C) 3.46; D) 3.87

**10** The exact value of $\int_0^1 \left(\frac{2x^3}{x^4+1}\right) dx$ is

A)$\frac{1}{2}ln5$; B) $2(ln2 - 1)$; C) $2ln2$; D)$\frac{1}{2}ln2$

**11** If $1 \leq |x - 2| \leq 4$ then

A) $3 \leq x \leq 6$; B) $x \leq 1$or $x \geq 3$;

C) $1 \leq x \leq 3$; D)$-2 \leq x \leq 10 \cup 3 \leq x \leq 6$

**12** Given that $1 < x < 2$ the exact value of $\sqrt{(X-1)^2} + \sqrt{(3-X)^2}$ is:

A) 4; B) 2; C) 3; D) 2x-4

**13** If $\sin(x+y) = x$, then $\frac{dy}{dx} =$

A) $\frac{1-\cos(x+y)}{\cos(x+y)}$; B) $\frac{1+\cos(x+y)}{\cos(x+y)}$

C) $\frac{\cos(x+y)+2}{\cos(x+y)}$; D) $\frac{\cos(x+y)-1}{\cos(x+y)}$

**14** The graph of $(X^2-1)Y = X^2-4$ has:

A) 1 horizontal & 1 vertical asymptotes
B) 2 vertical & no horizontal asymptotes
C) 1 horizontal & 2 vertical asymptotes
D) 2 vertical & 2 horizontal asymptotes

**15** If $a^2 - b^2 = 21$ and $a^2 + b^2 = 29$ where a and b are greater than 0, then ab is

A) -10; B) 5 √2; C) 10; D) -8

**16** A triangle with vertices (u,0), (v,8) and (0,0) has area

A) 4|u|; B) 2|v|; C)|uv|; D) 2|uv|

**17** The ellipse $4x^2 + 8y^2 = 64$ intersect the circle $x^2 + y^2 = 9$ at the point whose y coordinate is

A) +√2; B) +√5; C) +√6; D)+ √7

**18** The range of values of k for which the roots of the equation $kx^2 + 4x + k = 0$ are real and equal is: A) $0 < k < 2$ ; B)$|k| < 2$
C) $|k| > 2$; D) $-2 < k < 0$ or $0 < k < 2$

**19** If $|x - y| \leq |y - x|$ then:

A) $x < y$; B) $y > x$; C) $x > 0$ and $y > 0$:

D) true for all values of x and y

**20** If $x^2 - kx + k$ is exactly divisible by $(x-K)$ then the value of k is

A) 0; B) 0 or $\frac{1}{2}$
C) 1; D) No value

**21** $f(x) = \sqrt[3]{x}$ and $g(x) = x^3 + 8$ then fog(3) is

A) 33; B)5; C) 11; D) 35.0

**22** If n is a positive integer the remainder when $3x^{(2n+3)} - 4x^{(2n+2)} + 5x^{(2n+1)} - 8$ is divided by $(x-1)$ is: A) -20; B) -4; C) 10: D) 3

**23** The first three terms in the binomial expansion of $\left(4x + \frac{3}{x^2}\right)^{21}$ are

A) $1+4x+16x^2$; B) $1-4x+16x^2$ ,

C) $3+12x+48x^2$; D) $3-12x+48x^2$

**24** If P(M/N) = O, then

A) M and N are collectively exhaustive:
B) M and N are mutually exclusive;
C) M and N are complementary;
D) $P(M \cup N) = P(N)$

**25** The relationship between two variables $x$ and $y$ is expressed by the values in the table below.

| $x$ | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $y$ | 0.693 | 1.099 | 1.386 | 1.609 |

Using Trapezium rule, an approximate value for $\int_2^5 ydx$ is:

A)3.636; B) 2.394; C) 4.787; D) 3.663

## II- SHORT ANSWER QUESTIONS

**1** Given that the quadratic equation $(a^2+b^2)x^2-2(ab+bc)x+b^2+c^2=0$ has equal roots ,show that a, b and c are in geometric progression ,where a, b and c are non –zero real numbers. 5mks

**2** A password consists of five different letters of the English alphabet. Each letter may be used only once. How many passwords can be formed if :

i) all the letters of the alphabet can be used
ii) the password must start with a “D” and must end with an “L”. 2mks

**3** Consider the system of equations: $\begin{cases} x+y=3 \\ 3xy-z^2=9 \end{cases}$

i) How many solutions does the system of equations have in the set of real numbers? Justify your answer. 5mks

ii) Hence solve the system of equations.

**4** i)Solve for $x$, the equation $\log_x 8=1.5$.

ii) Evaluate $\dfrac{1}{\left[\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)\right]^6}$ 4mks

**5** Six out of twelve beads are to be threaded onto a ring. In how many ways can this be done? 3mks

**6** It is known of a large batch of cocoa seeds to be planted that for every three that will germinate, there is one seed that will not germinate. Five of these seeds are planted. Find the probability that only two will germinate. Leave your answer as a proper fraction. 3mks

**7** Given that $\theta$ is measured in radians and so small that $\theta^2$ and higher powers of $\theta$ are negligible, simplify $2\cos(\frac{\pi}{3}+\theta)$ , leaving your answer in surd form. 3mks

**8** Solve the equation: $\sqrt{x^2+4x+4}=x^2+5x+5$ 5mks

**9** After the first year the bank account of Mr peter decreased by 25%,during the second year it increased by 20%,during the third year it decreased by 10% and during the fourth year it increased by 20%.Does the account of Mr peter increase or decrease during these four years and by how much? 3mks

**10** Solve the differential equation $\sec x\frac{dy}{dx}=y$ , leaving your answer in the form $y=f(x)$ . 3mks