

MINISTERE DES ENSEIGNEMENTS SECONDAIRES

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MINISTRY OF SECONDARY EDUCATION

GBHS NYALLA
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|--------------------------|--------------|----------------------------|
| NAME OF STUDENT | | Date : |
| N° | | Classe : USSUSA |
| TEST N° : | PERIOD | DURATION : Three (3) Hours |
| EXPECTED OUTCOME : | | |

APPRECIATION OF COMPETENCE (MARK A CROSS(X))

| | NOT ACQUIRED (NA) | BEING ACQUIRED (EA) | ACQUIRED (A) |
|-------------------|-------------------|---------------------|--------------|
| EVALUATION MARK : | Paper 1 | | FINAL MARK |
| | Paper 2 : | | |
| | Paper 3 : | | |

PARENT : Name:

Date : Tel : Signature :

| | |
|--------------------------------|--|
| OBSERVATIONS (PARENT) : | |
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MATHEMATICS

Time Allowed: 3 hours.

Instructions:

1. This paper is arranged in three sections; A, B and C. Candidates are required to answer **ONLY**, **SECTIONS A and B**, or **SECTIONS A and C**. Candidates should **ANSWER EACH SECTION ON SEPARATE SHEETS**.
2. Sufficient working must be done to demonstrate the mathematical method employed. The number of marks is given in brackets at the end of each question or part-question. You are reminded of the necessity for good English and orderly presentations in your answers.

SECTION A: (Pure Mathematics)

1 Hour 30 Minutes

1. A function f is defined by $f(x) = 2x^3 - 3x^2 - 36x + 12$.
 - (i) Evaluate $f(3)$ and $f(-2)$.
 - (ii) Obtain $f'(x)$.
 - (iii) Use your expression for $f'(x)$ to find the coordinates of the two stationary points on the curve.
 - (iv) By drawing a sign table, distinguish between the stationary points.
 - (v) Obtain the point of inflexion.
 - (vi) Sketch the graph of $y = f(x)$. Deduce the range of values of k for which the equation $f(x) = k$ has three real roots.

[1+1+2+3+2+2=11 Marks]

2. (a) A function f is defined by $f(x) = |x^2 - 1| - |x^2 - 4|$, determine $f'(x)$.

- (b) Given that $\sin y = 2 \sin x$, show that

- (i) $\left(\frac{dy}{dx}\right)^2 = 1 + 3 \sec^2 y$.

- (ii) By differentiating (i) with respect to x , show that $\frac{d^2y}{dx^2} = 3 \sec^2 y \tan y$ and hence that,

$$\cot y \frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 + 1 = 0.$$

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[2+(2+3)=7 Marks]

3. (a) State, without prove, the following theorems;

- (i) Rolle's Theorem
- (ii) Mean Value Theorem

- (b) (i) Given the function $f(x) = x^2 - 2x - 3$ defined on the interval $[-1, 3]$. Determine whether the hypothesis of Rolle's theorem holds for the function f defined on the given interval and if they do, verify the conclusion of the theorem.

- (ii) Given that the function $f(x) = x^3$ is differentiable in the interval $(-2, 2)$, use the Mean Value theorem to find the value of x for which the tangent to the curve is parallel to the chord through the points $(-2, -8)$ and $(2, 8)$.

[(1+1)+(3+3)=8 Marks]

4. (a) Use the trapezium rule with five ordinates to estimate $\int_0^{0.8} e^{x^2} dx$.

(b) Solve the differential equation $x(1-y)\frac{dy}{dx} = -2y$ given that $y = 2$ when $x = e$, hence, show that $y = \ln\left(\frac{x^2 y}{2}\right)$.

5. (a) [3+3=6 Marks]

| | | | | | |
|---|------|-------|-------|--------|--------|
| x | 1.7 | 2.3 | 3.2 | 4.3 | 5.9 |
| y | 28.7 | 115.0 | 525.0 | 2100.0 | 8800.0 |

The table shows corresponding values of x and y obtained experimentally. It is given that x and y are connected by a relation of the form $y = ax^b$, where a and b are constants. By drawing a suitable linear graph, estimate the values of a and b to one decimal place.

(b) Show that the equation $f(x) = 0$, where $f(x) \equiv x^3 + x^2 - 2x - 1$, has a root in the interval $1 < x < 2$. Use the Newton-Raphson procedure, with initial value 1, to give two further approximations to the positive root of $f(x) = 0$, giving your answer to 2 decimal places.

[5+3=8Marks]

SECTION B: (Mathematics with Mechanics)

1 Hour 30 Minutes

1. The position vector of a particle of mass 5kg at time t seconds is r where

$$r = \left[(2 + t^2)i + tj + \left(3 - \frac{t^3}{3} \right)k \right] \text{m.}$$

- (a) Find the momentum and kinetic energy of the particle at time $t = 2$.
 (b) Find the work done by the particle between $t = 0$ and $t = 2$.
 (c) Show that the Cartesian equation of the path of the particle is $y(x-2) + 3z = 9$.

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[6+3+4=13 Marks]

2. A small smooth sphere P of mass 4kg , moving with speed 5ms^{-1} on a smooth horizontal plane strikes a second smooth sphere Q of equal radius but of mass 3kg , moving in the same direction with speed 2ms^{-1} . Given that the coefficient of restitution between the spheres is $\frac{1}{3}$,

- (a) calculate the loss in kinetic energy after impact,
 Given also that the spheres collide just at the end of the plane which is 6m above a horizontal floor, and that the subsequent motion is under gravity,
 (b) find the time taken by each sphere to strike the floor after impact.

[7+3=10 Marks]

3. (i) A car of mass 5 tonnes is moving along a horizontal straight road at a steady speed of 10ms^{-1} . The magnitude of the force resisting the motion of the car is proportional to the speed of the car.

(a) Given that the engine of the car is working at $40kW$, find the magnitude of the force resisting the motion of the car.

(b) The car ascends a hill whose inclination to the horizontal is θ , where $20 \sin \theta = 1$ at a steady speed of $5ms^{-1}$. Calculate the rate at which the engine of the car is working at this instance.

(ii) A uniform ladder ST of length $2a$ and weight W rest with its end S on a smooth horizontal floor and the end T against a smooth vertical wall. It is maintained in equilibrium at an angle $\arctan 2$ by a horizontal force P applied at S . Find the magnitude of P .

[(3+4)+6=13 Marks]

4. The two forces $F_1 = (-3i + 2j + k)N$ and $F_2 = (j - k)N$ act at points $r_1 = (2i - 3j + k)m$ and $r_2 = (-i + k)m$ respectively.

(a) Find the moment of F_1 about the origin.

(b) Show that the lines of action of F_1 and F_2 intersect.

A third force, F_3 , is added to the system. Given that the system is in equilibrium,

(c) find the magnitude of F_3 and a vector equation of its line of action.

[2+5+5=12 Marks]

(i) The events A and B are such $P(A/B) = \frac{7}{10}$, $P(B/A) = \frac{7}{15}$ and $P(A \cup B) = \frac{3}{5}$. Find,

(a) $P(A \cap B)$

(b) $P(A/B')$

(ii) When a car owner needs her car serviced, she phones one of three garages, X or Y or Z . Of her calls to them, 30%, are to garage X , 10% to Y and 60% to Z . The percentages for the occasions when the garage phoned can take the car in on the day of phoning are 20% for X , 6% for Y and 9% for Z .

(a) Find the probability that the garage phoned will not be able to take the car in on the day of phoning.

(b) Given that the car owner phones a garage and the garage will take her car in on that day, find the probability that she phones garage Y .

[(3+3)+(3+3)=12 Marks]

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EVERY NEXT LEVEL OF YOUR LIFE WILL DEMAND A DIFFERENT YOU.