REPUBLIQUE DU CAMEROUN
Paix – Travail – Patrie

MINISTERE DES ENSEIGNEMENTS SECONDAIRES

DELEGATION REGIONALE DE L'OUEST

INSPECTION REGIONALE DE PEDAGOGIE
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REPUBLC OF CAMEROON
Peace – Work - Fatherland

MINISTRY OF SECONDARY EDUCATION

REGIONAL DELGATION FOR THE WEST

REGIONAL INSPECTORATE OF PEDAGOGY
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F. MATHS 775 **West General Certificate of EDUCATION MOCK EXAMINATION**

March 2019 **ADVANCED LEVEL**

THE REGIONAL DELEGATION OF SECONDARY EDUCATION IN ASSOCIATION WITH THE **TEACHERS' RESOURCE UNIT**	SUBJECT CODE NUMBER **775**	PAPER NUMBER 3
MAHTEMATICS TEACHERS' PEDAGOGIC GROUP	SUBJECT TITLE **Further mathematics**	

Two and a half hours

INSTRUCTIONS TO CANDIDATES

Answer ALL questions

For your guidance, the approximate mark allocation for parts of each question is indicated in brackets.

Mathematical formulae and tables, published by the Board and noiseless non-programmable electronic calculators are allowed.

In calculations, you are advised to show all the steps in your working giving your answer at each stage.

Calculators are allowed.

Start each question on a fresh page.

1. Find as a series of ascending powers of x, up to and including the term in x^4, an approximate solution to the differential equation $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+8y=x$;

given that $y = 1$ and $\frac{dy}{dx}=0$ when $x = 0$. **(5 marks)**

Hence, using the approximations $h^2\left(\frac{d^2y}{dx^2}\right)_n \approx y_{n+1}-2y_{n-1}$ and $2h\left(\frac{dy}{dx}\right)_n \approx y_{n+1}-y_{n-1}$,

deduce that

$$(1+3h)y_{n+1}\approx(3h-1)y_{n-1}+\left(2-8h^2\right)y_n+h^2x_n.$$ **(3 marks)**

Using a step length of 0.2, show that

$$y_{n+1}\approx 1.05y_n-0.25y_{n-1}+0.25x_n.$$ **(1 mark)**

Find the value of y when $x = 0.6$. **(5 marks)**

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2. (i) Three forces $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, act at points with position vectors $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$, respectively where

$\mathbf{F}_1 = (3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ N, $\mathbf{r}_1 = (\mathbf{j} - 3\mathbf{k})$ m

$\mathbf{F}_2 = (\mathbf{i} - 4\mathbf{j} + \mathbf{k})$ N, $\mathbf{r}_2 = (3\mathbf{i} - 8\mathbf{j} + \mathbf{k})$ m

$\mathbf{F}_3 = (\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ N, $\mathbf{r}_3 = (2\mathbf{i} - \mathbf{j} + \mathbf{k})$ m.

When a fourth force $\mathbf{F}_4$ is added the system is in equilibrium.

(a) Find $\mathbf{F}_4$ and its position vector. **(1 mark)**

(b) Find also the magnitude of the moment of $\mathbf{F}_4$ about the origin. **(6 marks)**

(ii) Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on a particle moving it from a point with position vector $\mathbf{a}$ to a point with position vector $\mathbf{b}$. Given that

$\mathbf{F}_1 = (9\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ N, $\mathbf{F}_2 = (-\mathbf{i} + 15\mathbf{j})$ N

$\mathbf{a} = (2\mathbf{i} - \mathbf{j} + 4\mathbf{k})$ $\mathbf{b} = (9\mathbf{i} - 4\mathbf{j} + 7\mathbf{k})$.

Find the total work done on the particle. **(3 marks)**

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3. Two uniform smooth spheres A and B of equal radius are moving on a smooth horizontal plane. Sphere A has mass 2 kg and velocity $(2\mathbf{i} + \mathbf{j})$ m/s and sphere B has mass 3 kg and velocity $(\mathbf{i} - \mathbf{j})$ m/s. The spheres collide when the line joining their centres is parallel to $\mathbf{i} - \mathbf{j}$. Given that the coefficient of restitution between the two spheres is ½,

(a) Calculate, in vector form, the velocities of A and B immediately after the impact.

(b) Calculate the loss in kinetic energy due to the impact. **(10, 2 marks)**

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4. A particle moves so that at time t, its polar coordinates (r,θ) with respect to a fixed origin O are such that $r=\frac{a}{2+3\cos\theta}$, where a is a constant.

(a) At the point A, the value of r is a maximum.

Show that at this point, $r=\frac{a}{5}$. **(3 marks)**

(b) Show that $\dot{r}=\frac{3r^2}{a}\dot{\theta}\sin\theta$. **(3 marks)**

(c) Show that $\ddot{r}=\frac{3a}{25}\dot{\theta}^2$ when the particle is at A. **(4 marks)**

(d) The radial acceleration of the particle is $-\frac{\lambda}{12}$, where λ is a constant.

Find the speed of the particle at A in terms of λ and a. **(5 marks)**

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5. (i) Under certain circumstances, a raindrop falls so that its acceleration is $\left(g - \frac{1}{2}v\right)$ m/s^2 where v m/s is the velocity of the raindrop and g m/s^2 is the acceleration due to gravity. If the raindrop starts from rest, what time will elapse before the velocity is $2g$ m/s? **(3 marks)**

(ii) A body of mass m kg moves along a straight line against a constant resistance of magnitude $m\left(\lambda + \frac{v^2}{k}\right)$, where v m/s is the speed, and λ and k are positive constants.

Initially, the body is moving with speed u.

Show that the body comes to rest after covering a distance of $\frac{k}{2}\ln\left(1 + \frac{u^2}{\lambda k}\right)$, **(4 mks)**

and that the time taken is $\sqrt{\frac{k}{\lambda}}\tan^{-1}\left(\frac{u}{\sqrt{\lambda k}}\right)$. **(4 marks)**

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6. (i) The random variable X has probability density function

$$f(x) = \begin{cases} \frac{1}{4} & , 0 \le x \le 2 \\ \frac{1}{4}(2x + k), & 2 < x \le 3 \end{cases}$$

Find:

(a) the value of k. **(2 marks)**

(b) the cumulative distribution function F (x). **(4 marks)**

(c) the median. **(2 marks)**

(ii) The masses of packages from a particular machine are normally distributed with a mean of 200 g and a standard deviation of 2 g. Find the probability that a randomly selected package from the machine weighs

(a) less than 197 g. **(2 marks)**

(b) more than 200.5 g. **(2 marks)**

(c) between 198.5 g and 199.5 g. **(2 marks)**

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7. A particle P of mass m kg moves in a straight horizontal line. At time t seconds, the displacement of P from a fixed point O on the line is x m and P is moving with velocity $\dot{x}$ m/s. Throughout the motion two horizontal forces act on P:

One of the forces of magnitude $4mn^2|x|$ N is directed towards O, and the other is a resistance force of magnitude $2mk|\dot{x}|$ N, where n and k are positive constants.

(a) Show that $\ddot{x} + 2k\dot{x} + 4n^2x = 0$. **(2 marks)**

(b) In one case $k = n$ when $t = 0$, $x = a$ and $\dot{x} = 0$.

(i) Show that $x = e^{-nt}\left(a\cos\sqrt{3}nt + \frac{1}{3}\sqrt{3}a\sin\sqrt{3}nt\right)$. **(5 marks)**

(ii) Show that P passes through O when $\tan\sqrt{3}nt = -\sqrt{3}$. **(2 marks)**

(c) In a different case, $k = 2n$.

(i) Find the general solution for x at time t seconds. **(3 marks)**

(ii) Hence, state the type of damping which occurs. **(1 mark)**

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8. (i) Prove that the moment of inertia of a uniform rod of length $2a$ about an axis intersecting the rod at right angles at a distance b from its centre is $M\left(\frac{1}{3}a^2 + b^2\right)$, where M is the mass of the rod. **(4 marks)**

(ii) A thin uniform rod of length $2a$ and mass M attached to a smooth fixed hinge at one end O is allowed to fall from a horizontal position. Show that in the subsequent motion $2a\left(\dot{\theta}\right)^2 = 3g\sin\theta$,

where θ is the angle made by the rod and the horizontal. **(5 marks)**

(iii) A particle of unit mass moves in space so that at time t seconds, its position vector $\mathbf{r}$ is given by $\mathbf{r} = \left(\mathbf{i} + t^2\mathbf{j} + \cos\pi\mathbf{k}\right)m$.

Find its angular momentum about the origin when $t = 2$ s. **(2 marks)**